Hot spot identification by sensitivity analysis and probabilistic inference methods: Demonstration exercise

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Abstract

The method for identifying the presence of hot spots in the core of a nuclear reactor described in a previous note (Gandini, 2011) is applied in a demonstration exercise relevant to a simplified, medium size PWR reactor. By this method the information obtained on-line through a system of neutron measuring devices such as self-powered neutron detectors (SPNDs, or collectrons) inserted in the core of a nuclear power reactor allows the on-line quick detection of a possible hot spot during plant operation. The method is based on the generalized perturbation method (GPT) techniques for the calculation of the sensitivity coefficients of the integral quantities measured with the collectrons with respect to parameters representative of the hot spot, and on the use of probabilistic inference techniques, taking into account the errors associated with the measurements. The methodology allows to assess the effect on the quality of the hot spot detection system following possible failures of the measuring devices during the core life cycle. Such an assessment may be useful for defining an adequate protection strategy in terms of quality, number and distribution of the collectrons. The results obtained with the exercise demonstrate the validity of the proposed method.

1. Introduction

One of the problems posed during the operation of a nuclear reactor is the occurrence within the core of "hot spots", i.e., points where the ratio between the linear power density of the fuel element and its average value at nominal conditions exceeds an assigned threshold. For their quick on-line identification within a safety protection system, it has been proposed to exploit in-core devices called self-powered neutron detectors (SPNDs), or 'collectrons' (Mourlevat, 2001; Kroon, 1979). Such devices, once properly calibrated at nominal conditions, measure at real time in a number of different points of the core during the reactor operation the neutron capture rate in a given material (e.g., rhodium, or cobalt). This information may then be used, through an on-line algorithm for identifying the presence and localization of a potential hot spot. The method is based on the generalized perturbation method (GPT) techniques (Gandini, 1967, 1987, 1997) for the calculation of the sensitivity coefficients of the integral quantities measured with the collectrons with respect to parameters representative of the hot spot, and on the use of probabilistic inference techniques, taking into account the errors associated with the measurements. To note that with this method in a real case no energy/geometry simplification of the model assumed for the reference neutron flux calculation would be required. So, the presence of control elements, distributed burnable poisons, or zones with different enrichments and burnups could be accommodated without difficulty.

The protection associated with these SPND devices may be limited by the malfunctioning of one or more collectrons. It is then important to be able to quantify the increase of uncertainty on the hot spot value estimate in case one or more devices fail, so that it may be possible to determine whether, in some circumstances, the uncertainty affecting the dispersion range of the estimated values of the linear power density in the potential hot spot position makes its upper limit exceed pre-defined thresholds. In a previous note (Gandini, 2011) a method has been proposed to address this crucial problem. In the present article a demonstration exercise is illustrated relevant to a simplified, medium size PWR reactor. The results obtained demonstrate the validity of the method.

2. Definitions

Let us assume that a fixed number (N) of collectrons are positioned in the core of a given light water reactor. Let us assume then
that in case of an off-normal detection signal, there are M potential positions where a hot spot may occur. To simplify the presentation of the method, without limiting, as will become clear in the following, its extension to more complex cases, we assume that the core is represented in two-dimensional (x,y) geometry.

Let us assume for the moment the hypothesis that at all conditions, nominal or not, for each index m the value of the ratio
\[ r_m \equiv \frac{p_{\text{max},m}}{p_n} \]  
between the maximum linear power density and the average linear power density at each position m, is constant.

Let us denote by \( p_{\text{max},m} \) a first threshold for the linear power density value relevant to the mth of the M potential positions considered, above which an attention warning will be triggered, and a second threshold \( p_{\text{max},2} \) above which a plant shutdown would take place.

From the analysis of the measurements \( Q_n \) given by the collectors, the possibility of the presence, or not, of an hot spot condition in one of the M potential positions considered has to be evaluated and assessed in relation to the assigned thresholds. As a first step, the neutron flux distribution in multigroup vector form (\( \Phi \)) would be calculated, normalized on the basis of the overall assigned power. The following quantity is then considered:

\[ Q_n = <\Sigma^T \Phi>_{\text{collector}} \]  
i.e., the capture reaction rate of the detector within the generic collector n (\( \Sigma \), being its relevant multi-group macroscopic capture cross-section). A neutron source (‘external’, that is, not associated with fission reactions in the fuel) is then considered in correspondence to each of the M potential fuel element positions considered, given by the expression

\[ S_m \Sigma^T_c(m) \]  
where \( S_m \) is a scalar quantity, \( \Sigma^T_c \) a vector representing the fission neutron spectrum and \( \Sigma^T_c(m) \) a function equal to one inside the volume associated the mth potential hot spot position and equal to zero outside. In relation to the effect on the collectors, introducing an external source with a fission neutron spectrum is equivalent to considering a hypothetical accidental event within the fuel element considered that produces an equivalent increase of the fission (this time, ‘internal’) source.

At this point we define the sensitivity coefficients

\[ W_{mn} = \frac{dQ_n}{dS_m} \]  
Imposing that the power level is maintained constant, according to the generalized perturbation theory (GPT) (Gandini, 1987, 1997) these coefficients can be calculated using the expression

\[ W_{mn} = <\Phi^*_m \Sigma^T_c(m)> \]  
where \( \Phi^*_m \) the importance function associated with functional \( R_m \) results governed by the equation

\[ B \Phi^*_m + V \Sigma^T_c = Q_n \Sigma^T_c \]  
\( B \) being the overall fission rate, \( B \) the operator governing the neutron flux and \( \Sigma^T_c \) a function equal to one inside the volume of the detector material (or an equivalent volume in which it might have been homogenized) of the nth collector and zero outside.

3. Localization of the potential hot spot position

Each sensitivity coefficient \( W_{mn} \) defined by Eq. (5) represents the contribution of a unit fission source at position m to the reaction rate measurement in collector n. These coefficients form then a vector \( W_m \equiv [W_{mn}] \) characteristic of each potential hot spot position m.

Given a series of detections \( Q_n^m \), to which a corresponding vector \( Q_{\Sigma}^m \equiv [Q_{\Sigma}^m]^T \) may be associated, the search of a potential hot spot will start once the values of one or more components \( R_m \) depart significantly (i.e., beyond given uncertainty margins) from the nominal values \( R_{nm} \). In this case the candidate positions will be chosen among those for which the distribution of components of vector \( W_m \) will be closer to the distribution of components of vector \( Q^m - Q_n \), where \( Q_n \equiv [Q_{n,1}^m, . . . , Q_{n,M}^m]^T \), i.e., among those \( (say, M') \) for which, within a fixed range of uncertainty based on measurement accuracies, is minimal the sum

\[ S_m = \sum_{n=1}^N [x_1 (Q_{\Sigma,n}^n - Q_{n,1}) - x_{2,m}W_{mn}]^2 \]  
where \( x_1 \) and \( x_{2,m} \) are normalization coefficients, i.e.,

\[ x_1 = \frac{1}{\sum_{n=1}^N (Q_{\Sigma,n}^n - Q_{n,1})} \quad x_{2,m} = \sum_{n=1}^N W_{mn} \]  
A criterion to be used for determining the set of the \( M' \) candidate positions could be that of: first, identifying the position \( m \) such that \( S_m \) corresponds to the minimum sum and, secondly, selecting those position for which the sum

\[ S_m \leq (S_m + e_{sm}) \]  
where \( e_{sm} \) is the standard deviation (or a multiple of it, to be more conservative) of the sum \( S_m \), given by the expression

\[ e_{sm} = 2 \left[ \left( \sum_{n=1}^N x_1 (Q_{\Sigma,n}^n - Q_{n,1})^2 \right)^{1/2} \right]^{1/2} \]  
where \( e_{sm} \) is the experimental error associated to the nth detection.

4. Statistical analysis

We assume that in presence of a potential hot spot, this is (conservatively) attributable to a single position among those, \( M' \), identified by using the methodology described above. For each of these positions, we may proceed for the estimate of the most probable value \( s_{m'} \), and of the uncertainty associated with it. According to the "maximum likelihood" method of the statistical analysis, or equivalent approaches based on Bayesian inference principles, given a set of differences \( \Delta \Sigma \), between values measured on-line during operation and values at nominal (‘calculated’) conditions, and associated experimental errors \( e_{sm} \), we can determine the most probable value \( s_{m'} \), accounting for these differences (see Appendix A). Maintaining, for simplicity, the 2D (xy) geometry approach, to this value \( s_{m'} \) a corresponding difference of the average linear power density \( \Delta \rho_{av} \) can be associated on the basis of the equation

\[ \Delta \rho_{av} = \frac{\gamma}{L_f} \]  
where \( \gamma \) is the fission rate change within the volume associated with position \( m' \), \( \rho \) the energy units per fission and \( L_f \) the core active length. On the basis of Eq. (1), we may write the estimated value
\[ \bar{p}_{\text{max}} = r_m(p_{\text{weibull}} + \Delta p_{\text{weibull}}). \] (12)

To this value an uncertainty, \( \tilde{\varepsilon}_{\text{weibull}} \), will be associated, corresponding to the variance estimate given by Eq. (A.29).

5. Hot spot estimation

The highest among the \( M \) sums \( \bar{p}_{\text{max}}^i + \tilde{\varepsilon}_{\text{weibull}} \) would be finally confronted with the values \( p_{\text{max}}^{i1} \) and \( p_{\text{max}}^{i2} \) for assessing the decision to be taken. Such decision will then depend both on the linear power and on the associated standard deviation estimate. Rather than a single one, two or three standard deviations might be summed, for a more conservative approach.

5.1. Collectron system degradation

The above methodology allows also to take into account the detection degradation following collectron failures. To show this, let us assume that the collectron average annual rate failure (and thus the average time to failure) is known. As a consequence, during a core life cycle an average number of failure events would occur. Assuming that these events follow a given probability distributions, for example, an exponential one\(^2\) in case spurious events, independent of their time of operation, are considered, or a Weibull one\(^3\) in case the aging of the instrumentation has to be taken into account, we may obtain different random failure events histories according to the assigned distributions.

Let us choose a few failure histories during the core life cycle. For each history the collectrons that break down are subsequently excluded.

Consider a first history and assume that in a point \( n \), randomly selected among those chosen by a first screening, an event will occur leading to the formation of an hot spot, the presence of which would be simulated by a corresponding fictitious external source in the same position, with energy distribution corresponding to the fission spectrum. Through a numerical simulation, we calculate the perturbed (pseudo-experimental) values \( R_m^0 \) and perform a first evaluation exercise following the methods described above (with all the collectrons correctly functioning). Values \( \bar{\varepsilon}_{\text{weibull}} \) and, correspondingly, estimates \( \bar{p}_{\text{max}}^i \pm \tilde{\varepsilon}_{\text{weibull}} \) will be then obtained, which, if the number, arrangement and experimental accuracy assumed for the collectrons are adequate, would validate the presence of the assumed hot spot.

We repeat the exercise described above at the time of the first collectron failure, assuming that the event leading to the formation of the hot spot occurs immediately after it and in the same modalities as the first one. The failed collectron will be excluded from the new exercise. New estimates \( \bar{p}_{\text{max}}^{i\text{new}} \) will be obtained. The associated uncertainties \( \tilde{\varepsilon}_{\text{weibull}}^{\text{new}} \) resulting from following the same procedure will be now larger, given the reduced information available, this making the interpretation of results more difficult. We may then proceed to consider the second fault, and so on for the following ones.

With the degradation of the available information, the uncertainties of the resulting estimates will become gradually larger. With the growth of these uncertainties, the error margins which have to be taken into account will make the safety system progressively less viable, forcing it to an halt ahead of schedule. If the possibility of this occurrence is found to be beyond acceptable probability limits, a larger number of collectrons, or their different distribution or an increased frequency of their scheduled replacement, or some other improvement for making them more reliable should be considered.

The methodology could also allow a reduction of the number of collectrons if, following the same procedure, it turns out to be redundant.

6. Method generalization

So far we have assumed a system represented by a two-dimensional \((x,y)\) geometry. In the case of a three-dimensional \((x,y,z)\) geometry, a similar procedure would follow considering the arrangement of collectrons in radial and axial positions \( n, k \) \( (n = 1, 2, \ldots; N; k = 1, 2, \ldots, K) \), respectively. In each of these positions quantities \( Q_{\text{weibull}} \) analogous to those defined by Eq. (2) would be measured.

Similarly, the positions of hypothetical hot spots will be identified with radial and axial indices \( m, h \) \( (m = 1, 2, \ldots; M; h = 1, 2, \ldots, H) \). Sensitivity coefficients \( \omega_{\text{weibull}} \) will be then considered, corresponding to the contribution of a neutron fission at point \((m, h)\) to the detection at the collectron position \((n, k)\), defined by the equation

\[ \omega_{\text{weibull}}^{\text{sh}} = \langle \psi_{\text{weibull}}^T Z(n, k) \rangle >, \] (13)

where \( \tilde{\varepsilon}_{\text{weibull}}(r) \) is a function equal to one within the volume associated with the potential hot spot position \((m, h)\) and equal to zero outside it. To each unit neutron fission source placed in a position \((m, h)\) will correspond, in place of a characteristic vector as in the case of two-dimensional geometry, a characteristic matrix \((\text{of size } N \times K)\) of sensitivity coefficients. Following a procedure similar to that used in the two-dimensional case we can localize a few, most probable hot spot positions \((m', h')\).

Given a set of differences \( \Delta Q_{\text{weibull}} \) between measured and calculated values, with associated experimental errors \( \tilde{\varepsilon}_{\text{weibull}} \), it is possible also in this case, following a procedure similar to that described above, to localize and obtain estimates of potential hot spots.

7. Updating procedure

As seen above, for identifying the most probable position and strength of a possible hot spot, the methodology requires the pre-calculation of the sensitivity coefficients \( w_{\text{weibull}} \) defined by Eq. (4). These coefficients are subject to changes due to nuclide evolution during burn-up and/or control operation adjustments (e.g., boron concentration changes in the coolant water). So, in order to take into account these material density changes, and up to a point at which a full recalibration of the neutron flux and importance functions would result mandatory along with some established criteria (defined by an appropriate analysis of the errors associated with limiting to first order the perturbation expressions), the values of these sensitivity coefficients could be updated at burn-up predetermined time steps, or in cases of control material changes exceeding predefined levels, by simply replacing Eq. (5), which defines them along the GPT theory, with the equation

\[ w_{\text{weibull}}(t_i) = \langle \psi_{\text{weibull}}^T Z(r_m) \rangle > + < \psi_{\text{weibull}}^T \Delta B(t_i) \Phi >, \] (14)

\( \Delta B(t_i) \) being the change of the governing transport/diffusion operator following the material density changes occurring at a defined time \( t_i \). This updating procedure would not imply a significant time delay to the protection system response, the added calculations involved implying simply algebraic operations on ‘a priori’ known quantities for determining the terms at the right side of Eq. (14). In cases of substantial changes of the system, for which second or-
under terms could not be neglected, a new calculation of the importance functions, and consequently of the sensitivity coefficients, should be made. Adopting as first guess the functions calculated at the first step would reduce dramatically the number of iterations required for reaching convergence, thus correspondingly reducing the calculation time required.

8. Demonstrative numerical simulation

For a numerical simulation demonstration, a medium size PWR system, the MARS project (Cumo et al., 2004), has been considered. For the demonstration purposes of this study a bidimensional geometry has been considered adequate, as represented in Fig. 1 where its XY transverse section is shown.

The axial neutron leakage has been taken into account introducing a buckling term. The fuel elements form a square lattice, with each element formed by $17 \times 17$ fuel pins. The relevant specifications are given in Table 1.

The main simplification that has been made with respect to the original project is the absence of control rods. The ERANOS code (Rimpault et al., 2002) has been adopted for the analysis. The calculations were carried out in diffusion approximation using a 15 group cross-section library.

As regards the detector material in the collectrons, cobalt-59 has been chosen. The presence of this detector has been simulated by replacing in a fuel element 0.0436% vol of water with cobalt.

The simplifications taken are justified by the fact that in the exercises considered we are merely interested in testing the methodology. The validity of the results obtained should however be reasonably extrapolable to more realistic configurations.

In Fig. 2, the positions of the elements containing the collectron devices are indicated together with the positions of the fuel elements in which the occurrence of a potential hot spot have been considered (limited for simplicity to the first quadrant).

In Table 2, the sensitivity coefficients $w_{sn}$ defined by the Eq. (4) are reported. As mentioned before, they correspond to the increase in collectors 1, 3 and 4, as may be also verified by observing the values of sensitivity coefficients of Table 2 in the column corresponding to fuel position 8.

8.1. Exercise 1

Assume now the presence of a hot spot in the element of fuel corresponding to the position 8, as shown in Fig 3, simulated for simplicity by a neutron source equal to 1. Intuitively, we deduce that its presence will be detected by the larger capture rate increase in collectors 1, 3 and 4, as may be also verified by observing the values of sensitivity coefficients of Table 2 in the column corresponding to fuel position 8.

8.1.1. Identification of the hot spot candidates

For this simulation exercise, the ‘detectors’ $Q_{in}$ have been assumed corresponding to a set of quantities randomly sorted according to a Gaussian distribution law characterized by average values $Q_{out}$ and a 5% standard deviation. Adapting the method illustrated in Section 2, it is straightforward to identify the hot spot candidates, i.e., the positions in correspondence to which the sum defined by Eq. (7), within a given uncertainty range, is minimal. For illustration, in Table 3 the values of such sum are given for each fuel element position. It may be seen that the minimum value corresponds to position 8, as expected.

8.1.2. Probabilistic inference

Once the position of a possible hot spot candidates is identified, and the coefficients of the sensitivity coefficients related to it are determined, we may use the probabilistic inference methods (see Appendix A) for estimating the value of the neutron source, here viewed as a simulation of the fission neutron surge produced with the hot spot occurrence, and the statistical error associated with it.

For what concerns the set of $M$ possible candidates, $M$ being the number of candidates satisfying condition (9), in case all the collectrons are properly functioning, it results $M = 1$, only fuel cell positions 8 satisfying such condition. The probabilistic inference methodology used gives for it, along with Eq. (A.27), an estimate of 1.003. Whereas the hot spot reconstruction results independent of the neutron flux level normalization, only the sensitivity coefficients defined by Eq. (4) being involved in the calculation procedure, its error estimate depends on it. In this numerical simulation the assumed flux normalization leads to detection values $Q_n$ as given by Table 4. In the same table are also given the relative (percent) detections changes produced in the collectrons by the unit source at fuel element position 8 which in this exercise simulates the hot spot. It may be seen that the detection values at collectrons 1 and 3 are augmented by about 25%, at collectron 4 by about 14% whereas in the other collectrons they are reduced to negative values due the constant power constraint. Having then assumed a detection accuracy of 3%, the standard deviation associated with the hot spot estimate, along with Eq. (A.28), results of the order of 8%. So, the estimated, conservative quantity to be confronted with an hypothetical alert threshold in this numerical simulation would be (adding conservatively two standard deviations to the hot spot estimate) 1.16.

8.1.3. Error increase with collectron failures

Over a period of time, the collectrons may fail due to several reasons. In such cases, the failed collectron needs to be identified and the possible hot spot detection estimated by other “neighbor” detectors. The failed detector can then be replaced when the scheduled reactor shutdown (for maintenance, etc.) takes place.
To be able to perform the tasks of identifying faulty detectors, while maintaining more efficiently of the hot spot protection system, the collectors should be continuously monitored on line. In case the ratio of the measurement of a given collector with respect to the measurements of neighboring ones results significantly altered [along an established criterion, and having ruled out the presence of a hot spot], compared with the values at normal conditions, such collector should be excluded from the system. As will be seen in the following, up to a certain number of detector units, such exclusions do not seem to compromise significantly the efficiency of a protection system in which the methodology presented above is implemented.

In case one or more collectors fail, a loss of the information normally associated with them will occur and, as a consequence, the error associated with the source estimate reproducing the fission neutron source surge will become larger. In Table 5 it is
shown how the error of the neutron source estimate increases with the number of failures while the source estimate remains close to unity. We may also see that the number of the resulting candidates for hot spot positions in this case remains just one (coincident with that assumed in the simulation) up to five subsequent failures starting from the collectrons closest to the hot spot.

Just a word relevant to the accuracy associated with the calculated coefficients. A number of cases have been run showing that the inaccuracies of these coefficients, although resulting not having a significant impact on the standard deviation given by Eq.(A.29), they affect directly the hot spot estimate given by Eq.(A.27). So, for example, a 5% inaccuracy on the calculated coefficients would generally imply an added inaccuracy on the hot spot reconstruction of the same order.

8.2. Exercise 2

In this case the results are similar to those of the previous case. The source, at position 25, is close only to collectron 4, as shown in Fig. 4. As can be seen from Table 6 the hot spot position is easily identified.

8.2.1. Probabilistic inference

For what concerns the number $M$ of the set of probable candidates, in case all the collectrons are properly functioning, also

<table>
<thead>
<tr>
<th>Collectrons</th>
<th>$Q_n$</th>
<th>$\Delta Q_n/Q_n$ (produced by a unit fission source in fuel element at position 8) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.557</td>
<td>25.15</td>
</tr>
<tr>
<td>2</td>
<td>0.559</td>
<td>5.77</td>
</tr>
<tr>
<td>3</td>
<td>0.558</td>
<td>25.16</td>
</tr>
<tr>
<td>4</td>
<td>0.676</td>
<td>16.56</td>
</tr>
<tr>
<td>5</td>
<td>0.677</td>
<td>-6.09</td>
</tr>
<tr>
<td>6</td>
<td>0.558</td>
<td>-6.68</td>
</tr>
<tr>
<td>7</td>
<td>0.556</td>
<td>-5.81</td>
</tr>
<tr>
<td>8</td>
<td>0.674</td>
<td>-6.11</td>
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<tr>
<td>9</td>
<td>0.675</td>
<td>-6.55</td>
</tr>
<tr>
<td>10</td>
<td>0.557</td>
<td>-6.71</td>
</tr>
<tr>
<td>11</td>
<td>0.556</td>
<td>-6.74</td>
</tr>
</tbody>
</table>
for this case it results $M_0 = 1$, only fuel cell positions 25 satisfying condition (9). The probabilistic inference methodology used gives for it, along with Eq. (A.27), an estimate of 1.008. If the accuracy by which the collectron detection measurements are made is assumed to be 3%, the standard deviation associated with this estimate, along with Eq. (A.28), results of the order of 11%.

In this numerical simulation the assumed flux normalization leads to detection values $Q_n$ as given by Table 7. In the same table are also given the detections changes produced in the collectrons by the unit source in fuel element position 25 which in this exercise simulates the hot spot. It may be seen that the detection value at collectron 4 is augmented by up to 14% whereas in the other collectrons the detection values result reduced to negative values due the constant power constraint. Having assumed a detection accuracy of 3%, the standard deviation associated with the hot spot estimate, along with Eq. (A.28), results of the order of 11%.

### Table 6
Values of the sum defined by Eq. (7).

<table>
<thead>
<tr>
<th>Hot spot position</th>
<th>SUM</th>
<th>Hot spot position</th>
<th>SUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.697E+00</td>
<td>14</td>
<td>2.181E+00</td>
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<tr>
<td>2</td>
<td>3.655E+01</td>
<td>15</td>
<td>1.472E+01</td>
</tr>
<tr>
<td>3</td>
<td>3.447E+02</td>
<td>16</td>
<td>6.210E+00</td>
</tr>
<tr>
<td>4</td>
<td>5.474E+00</td>
<td>17</td>
<td>2.232E+02</td>
</tr>
<tr>
<td>5</td>
<td>3.088E+01</td>
<td>18</td>
<td>9.313E+00</td>
</tr>
<tr>
<td>6</td>
<td>8.035E+00</td>
<td>19</td>
<td>1.962E+00</td>
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<tr>
<td>7</td>
<td>4.787E+00</td>
<td>20</td>
<td>1.619E+00</td>
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<td>12</td>
<td>3.707E+01</td>
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<td>2.523E+01</td>
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<tr>
<td>13</td>
<td>1.463E+01</td>
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<td></td>
</tr>
</tbody>
</table>

### Table 7
Values $Q_n$ and the relative (percent) detections changes.

<table>
<thead>
<tr>
<th>Collectron</th>
<th>$Q_n$</th>
<th>$\Delta Q_n/Q_n$ (produced by a unit fission source in fuel element at position 25) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.557</td>
<td>5.14</td>
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<tr>
<td>2</td>
<td>0.559</td>
<td>6.90</td>
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<td>3</td>
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<td>5.06</td>
</tr>
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<td>14.34</td>
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<td>5</td>
<td>0.677</td>
<td>0.23</td>
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<tr>
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<td>0.20</td>
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</tr>
<tr>
<td>12</td>
<td>0.557</td>
<td>8.44</td>
</tr>
</tbody>
</table>

### Table 8
Results from simulation exercise 2 (hot spot simulated in position 25) (collectron detections assumed independent, each with 3% error).

<table>
<thead>
<tr>
<th>Failed collectrons</th>
<th>Hot spot candidate positions (satisfying condition, Eq. (9))</th>
<th>Hot spot (incidental fission neutron source surge)</th>
<th>Simulated $\Delta Q_n$</th>
<th>Estimated $\Delta Q_n$</th>
<th>Standard deviation $\Delta Q_n$</th>
</tr>
</thead>
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Fig. 4. Reactor layout hot spot position (in black) close to a single detector.
8.2.2. Error increase with collectron failures

Also for this case, as shown in Table 8, the error of the neutron source estimate increases with the number of failures. We may also see that the number of the resulting candidates for hot spot positions in this case remains just one (coincident with that assumed in the simulation) up to seven subsequent failures starting from the collectrons closest to the hot spot.

9. Conclusions

The results obtained with the above simulation exercises show how the methodology proposed in this work may be used with success as a hot spot identification tool by fully exploiting the information available from a collectron detection system implemented in a nuclear reactor plant.

This methodology may be useful also in a reactor design stage. An extensive analysis relevant to the collectron distribution and their failure sequences could allow, in fact, to identify optimal configurations based on plant engineering and economic considerations.

For what concerns the representation of the collectrons, we have described them in 2D geometry assuming a certain amount of 59Co dissolved in specified fuel elements. In reality, the collectrons are distributed in the core also along vertical channels.

The use of the GPT methodology, in conclusion, allows to construct an 'online' hot spot diagnostic system exploiting in an optimal way the information available from the collectrons.

To note that the calculations to be made are relatively fast. Once pre-calculated the importance functions required, the determination of the sensitivity coefficients with respect to any hypothetical hot spot position consists merely on a series of simple algebraic operations.

The methodology allows also to assess the effects of the detection system degradation due to possible collectron failures, and so to decide on their optimal number to be considered for the core life cycle and on their best distribution.

In the cases considered in above exercises we have assumed that the system continues to operate at constant power after the formation of an hot spot. The methodology can be still be used even in cases in which a sudden hot spot formation occurs such as to create a power transient detectable (apart from the measurements in cases in which a sudden hot spot formation occurs such as to

\[ Q_i = Q_0(p_1, p_2, \ldots, p_j). \quad (A.1) \]

If we assume a given set of values \( p_{0j} \) close to the true ones, we can expand Eq. (A.1). Neglecting terms above the first order, we will have

\[ Q_i = Q_0(p_1, p_2, \ldots, p_j) + \sum_{j=1}^{L} \frac{dQ_i}{dp_j} (p_j - p_{0j}). \quad (A.2) \]

The quantities \( Q_i \) calculated on the basis of the values of the parameters \( p_{0j} \) will be denoted as \( Q_i^{\text{cal}} \), i.e.,

\[ Q_i^{\text{cal}} = Q_0(p_1, p_2, \ldots, p_j) \quad (\ell = 1, 2, \ldots, L) \quad (A.3) \]

We define the quantities

\[ y_j = p_i - p_{0j} \quad (j = 1, 2, \ldots, J) \quad (A.4) \]

\[ y_{j,\ell} = Q_i - Q_i^{\text{cal}} \quad (\ell = 1, 2, \ldots, L) \quad (A.5) \]

\[ s_{j,\ell} = \frac{\partial Q_i}{\partial p_j} \quad (\ell = 1, 2, \ldots, L; \ j = 1, 2, \ldots, J) \quad (A.6) \]

where the sensitivity coefficients \( s_{j,\ell} \) are calculated through the GPT method. We introduce now the vectors

\[ y_p = \begin{bmatrix} y_1 \\ \vdots \\ y_J \end{bmatrix}, \quad y_q = \begin{bmatrix} y_{1,1} \\ \vdots \\ y_{J,1} \end{bmatrix}, \quad y = \begin{bmatrix} y_p \\ y_q \end{bmatrix} \quad (A.7) \]

and the matrix

\[ W = |S - U| \quad (A.8) \]

where \( U \) is a \( L \times L \) unit matrix, and \( S \) is the sensitivity matrix

\[ S = \begin{bmatrix} s_{1,1} & s_{1,2} & \cdots & s_{1,J} \\ s_{2,1} & s_{2,2} & \cdots & s_{2,J} \\ \vdots & \vdots & \vdots & \vdots \\ s_{J,1} & s_{J,2} & \cdots & s_{J,J} \end{bmatrix} \quad (A.9) \]

Eq. (A.2) can then be written in vector form

\[ W y_p = 0 \quad (A.10) \]

With this method it is proposed to obtain the best estimate of the parameters \( p_j \) based on the experimental information available, which we can define associated to the quantities

\[ y_{j,\ell}^{0} = p_{0j} - p_{j} \quad (j = 1, 2, \ldots, J) \quad (A.11) \]

\[ y_{j,\ell}^{0} = Q_{i}^{0} - Q_{i}^{\text{cal}} \quad (\ell = 1, 2, \ldots, L) \quad (A.12) \]

where \( Q_{i}^{0} \) are the integral values obtained experimentally, while \( p_{j}^{0} \) the values obtained either experimentally or via theoretical models, or other hypotheses\(^4\).

Quantities \( Q_{i}^{0} \) and \( p_{j}^{0} \) are generally associated with errors, or uncertainties estimates represented by dispersion matrices (or variance/covariance) \( C_0 \) and \( C_p \), respectively. We can then define the dispersion matrix:

\[ Q = \begin{bmatrix} C_0 & 0 \\ 0 & C_p \end{bmatrix} \quad (A.13) \]

\(^4\) The values indicated with \( p_{j}^{0} \) can be obtained either experimentally or via theoretical models, or other hypotheses. Along the Bayesian inference theory, the quantities \( p_{j}^{0} \) may be associated to the so called 'prior' information, while the quantities \( Q_{i}^{0} \) to the 'posterior' one.
in which the generic element \( c_{ik} \) is the expected value
\[
\bar{c}_{ik} = E[\langle y_{ex}^i \rangle y_{ex}^i] = \langle y_{ex}^i \rangle \langle y_{ex}^i \rangle
\]

The likelihood function for vector \( \mathbf{y} \) in this case will be defined by the expression
\[
L(\mathbf{y} | \mathbf{y}^a) = \frac{1}{(2\pi)^{L/2} |\mathbf{C}_p|} \exp \left\{ -\frac{1}{2} \mathbf{y}^T \mathbf{C}_p^{-1} \mathbf{y} \right\}
\]

This function is maximum, and then corresponding to values of \( \mathbf{y} \) with highest probability, if an estimator \( \mathbf{y} \) of \( \mathbf{y} \) is chosen such that the product
\[
(\mathbf{y}^a - \mathbf{y})^T \mathbf{C}_p^{-1} (\mathbf{y}^a - \mathbf{y}) = \text{minimum}
\]

with the constraints
\[
\mathbf{W} \mathbf{y} = \mathbf{0}
\]

A.1. Method of reduction by the Lagrange multipliers

Vector \( \mathbf{y} \), and then \( \mathbf{y}_p \) ed \( \mathbf{y}_q \), can be obtained by the method of Lagrange multipliers (Gandini, 1988). The estimate are given by the expression
\[
\mathbf{y} = \mathbf{y}^a + \mathbf{C}_p \mathbf{W}^T (\mathbf{W} \mathbf{C}_p \mathbf{W}^T)^{-1} \mathbf{W} \mathbf{y}^a
\]

and in particular, with respect to estimates \( \mathbf{y}_p \),
\[
\mathbf{y}_p = \mathbf{y}_p^a + \mathbf{C}_p \mathbf{S}_p^T (\mathbf{C}_p + \mathbf{SC}_p \mathbf{S}_p)^{-1} (\mathbf{y}_p^a - \mathbf{y}_p^c)
\]

If the starting values that define vector \( \mathbf{p}_0 \) are assumed numerically identical to those that define vector \( \mathbf{p}^a \), from Eq. (A.11) we have
\[
\mathbf{y}_p^a = \mathbf{0}
\]

and then Eq. (A.19) reduces to
\[
\mathbf{y}_p = \mathbf{C}_p \mathbf{S}_p^T (\mathbf{C}_p + \mathbf{SC}_p \mathbf{S}_p)^{-1} (\mathbf{y}_p^a - \mathbf{y}_p^c)
\]

The error dispersion matrix associated with quantities \( \mathbf{y}_p \) is then
\[
\mathbf{C}_p = \mathbf{C}_p + \mathbf{C}_p \mathbf{S}_p^T (\mathbf{C}_p + \mathbf{SC}_p \mathbf{S}_p)^{-1} \mathbf{SC}_p
\]

A.2. Method of reduction by elements

In the reduction method using the Lagrange multipliers the matrix that has to be inverted appearing in Eqs. (A.20) and (A.21), i.e., \((\mathbf{C}_p + \mathbf{SC}_p \mathbf{S}_p)^{-1}\), has the dimensions given by the number of integral quantities, namely \( L \times L \). In the event that the number of the integral quantities, \( L \) is larger than that of the parameters, the use of an alternative, equivalent method is preferable, known as the method by elements (Gandini, 1988). This method should in any case be used when the dispersion matrix \( \mathbf{C}_p \) is undefined and therefore its inverse has a singularity. Along this method the expression corresponding to (A.19) is the following
\[
\mathbf{y}_p = (\mathbf{S}^T \mathbf{C}_p \mathbf{S} + \mathbf{C}_p^{-1})^{-1} (\mathbf{C}_p^{-1} \mathbf{y}_p^a + \mathbf{S}^T \mathbf{C}_p^{-1} \mathbf{y}_p^c)
\]

and if we can assume that \( \mathbf{y}_p^a = \mathbf{0} \)
\[
\mathbf{y}_p = (\mathbf{S}^T \mathbf{C}_p \mathbf{S} + \mathbf{C}_p^{-1})^{-1} \mathbf{S}^T \mathbf{C}_p^{-1} \mathbf{y}_p^c
\]

The error dispersion matrix associated with quantities \( \mathbf{y}_p \) is then
\[
\mathbf{C}_p = \mathbf{C}_p + \mathbf{S}^T \mathbf{C}_p \mathbf{S}
\]

A.3. Application of the statistical reduction method for the potential hot spot identification

According to the method described in the preceding paragraphs for the identification of a potential hot spot occurring in a given reactor by in-core measuring devices (collectrons), the parameter considered for its simulation would be the intensity of a local external source characterized by a fission neutron spectrum and to be determined along with the probabilistic inference methodolog-
References


