LETTER TO THE EDITORS

ON THE PERTURBATION METHOD FOR BURNUP AND SHUFFLING ANALYSIS

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In a recent article (Gandini, 1987) a perturbation method for fuel evolution and shuffling analysis was proposed, making use of the heuristically-based GPT theory. To substantiate the formalism, a very simple illustrative example was considered, consisting of a single fissile nuclide (of density c) subject to burnup in an infinite, homogeneous medium, at a given power density and maintained at steady state condition by an intensive control variable \( \rho(t) \) (\( \rho \)-reset). A one energy group for the neutrons (of density \( n \)) was assumed. The field of interest is then \( f = \ln c \). The residual reactivity \( R(t_F) \), i.e. the reactivity associated with the control material at the end of cycle, was then considered, equation (65), i.e.

\[
R(t_F) = \left. \left( \Phi^* / \Phi c_0 \sigma_B \right) \right|_{t_F} = \left. \Phi^* c_0 \sigma_B \nu n \right|_{t_F} = \frac{1}{\sigma_f} \left[ \nu \sigma_f - \sigma_f - \Sigma_e \right] \left. c(t_F) \right|_{t_F},
\]

where \( \Sigma_e \) is the macroscopic capture cross-section of materials other than fuel and control.

The sensitivity of \( R(t_F) \) with respect to the initial nuclide density \( c_0 \) (for simplicity, limiting consideration to this parameter) results, from equations (64) and (65):

\[
\frac{dR(t_F)}{dc_0} = \frac{c_0}{c^2(t_F)\nu \sigma_f} (\Sigma_e + 2\sigma_f n) \approx \frac{c_0}{c^2(t_F)\nu \sigma_f} \Sigma_e \quad (\rho\text{-mode}).
\]

This expression gives the desired quantity, i.e. the first derivative at \( t_F \) of \( R \) with respect to \( c_0 \) (which, for the present case, can be easily obtained analytically). The extension of the field to the proper intensive control variable \( \rho(t) \) (multiplying the control material distribution (in the present case, a constant) and such that the power history is maintained) seems mandatory in those cases in which the control material appears in the very functional, or response, to be studied [as is the case with the residual reactivity \( R(t_F) \)].

If, instead, the average functional is considered:

\[
\bar{R} = \frac{1}{t_F - t_0} \int_{t_0}^{t_F} R(t) \, dt,
\]

the following sensitivity coefficients are obtained:

\[
\frac{d\bar{R}}{dc_0} = \frac{1}{c(t_F)\nu \sigma_f} \Sigma_e \quad (\rho\text{-mode})
\]

and

\[
\frac{d\bar{R}}{dc_0} = \frac{1}{t_F - t_0} \int_{t_0}^{t_F} \frac{dK_{at}(t)}{dc_0} \, dt
\]

assuming \( d\bar{R} \approx \int_{t_0}^{t_F} \frac{dK_{at}(t)}{dc_0} \, dt \).

Similar discrepancies between the \( \rho \)-mode and the \( \lambda \)-mode expressions are obtained if we consider other functionals, such as

\[
\int_{t_0}^{t_F} [R(t) - \bar{R}]^2 \, dt.
\]

The above discrepancies between the \( \rho \)-mode and the \( \lambda \)-mode approaches, as far as they imply the definition and time-dependence of the functions involved, may be considered at all general and, therefore, representative also of complex realistic cases. It may be concluded, then, that the more realistic \( \rho \)-mode GPT scheme seems more apt to guarantee a correct description of the real effects to be analyzed.

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REFERENCES: