ON THE EVALUATION OF ADS SUBCRITICALITY

A. Gandini

University of Rome, Nuclear Engineering Department (DINCE)
Corso Vittorio Emanuele II 224, 00186 Rome - Italy

Abstract

Basing on a recent derivation of perturbation methods generally applicable for the analysis of subcritical reactors, at steady state or transient conditions, a simple procedure is described for the evaluation of their subcriticality level during normal operation. The precision of the method is shown to increase with the system approaching criticality conditions.

A problem connected with the operation of subcritical (ADS) reactors is posed by the ability of evaluating with sufficient precision their subcriticality level. In this note we illustrate a general approach to this problem, making use of a recent derivation of the zero kinetics equations relevant to these systems (Gandini, 2001). These equations have been obtained starting from the transport, or diffusion ones governing the neutron and the i’th precursor densities and resulted, in terms of the normalized power $P$ and of the “effective” precursor density $\xi_i$,

$\ell_{\text{eff}} \frac{dP}{dt} = \left(\rho_{\text{gen}} - \alpha \beta_{\text{eff}}\right)P + \alpha \sum_{i=1}^{I} \lambda_i \xi_i + \zeta(1-P) + \rho_{\text{source}}$  \(1\)

$\frac{d\xi_i}{dt} = \beta_{i,\text{eff}} P - \lambda_i \xi_i$  \(2\)

where $\alpha$ is a coefficient taking into account the spectrum of the delayed neutrons, $\rho_{\text{gen}}$ is a “generalized reactivity”, $\rho_{\text{source}}$ is the reactivity associated with an external source change, and $\zeta$ a subcriticality index.

* For simplicity of notation, the normalized power $P$ is intended here to be the ratio $\frac{W}{W_o (1+q)} = \frac{\lambda <\Sigma_f, \phi_o>}{\gamma <\Sigma_{f,o}, \phi_o>}$, where $W_o$ is the power at unperturbed, nominal conditions, $\gamma$ is the number of energy units per fission, and where $\frac{<\delta \Sigma_f, \phi_o>}{\gamma <\Sigma_{f,o}, \phi_o>}$ is the reactivity associated with an external source change, and $\zeta$ a subcriticality index.
With the subcriticality decreasing, $\zeta$ tends to zero and the above equations to those relevant to a critical system.

The full expressions of the physically meaningful quantities appearing in Eqs. (1) and (2) are given in Appendix, together with a short (revised) presentation of the methodology used for their derivation. The generalized reactivity, in particular, is defined as

$$\rho_{\text{gen}} = \zeta \rho_{\text{gen,s}}, \quad (3)$$

i.e., by the product of the source strength change required to reset the power level altered by the perturbation, given by the expression

$$\rho_{\text{gen,s}} = <n_{s,0}^*, (\Delta A + \delta S_f)\phi_o^* > + \frac{\gamma}{W_0} <\delta \Sigma_f, \phi_o^* >, \quad (4)$$

multiplied by the subcriticality index

$$\zeta = \frac{1}{<n_{s,0}^*, \chi S_f \phi_o^* >} \equiv \frac{1 - K_{\text{sub}}}{K_{\text{sub}}}, \quad (5)$$

where

$$K_{\text{sub}} = \frac{<n_{s,0}^*, \chi S_{f,o} \phi_o^* >}{1 + <n_{s,0}^*, \chi S_{f,o} \phi_o^* >}. \quad (6)$$

Coefficient $K_{\text{sub}}$ merges into $K_{\text{eff}}$ (the multiplication coefficient relevant to the fundamental eigenfunction) with the system approaching criticality.

In the following, a method is described for determining experimentally the subcriticality level, making use of the above definitions.

Consider a change of a (calibrated) control rod position. This would correspond to an experimental reactivity value $\left(\delta K_{\text{eff}} / K_{\text{eff}}\right)^{\text{exp}}$. The associated value $\rho_{\text{gen,B}}^{\text{exp}}$ of the generalized reactivity could be assumed as

$$\rho_{\text{gen,B}}^{\text{exp}} = \rho_{\text{gen,B}}^{\text{cal}} \left(\delta K_{\text{eff}} / K_{\text{eff}}\right)^{\text{exp}}_{B} \left(\delta K_{\text{eff}} / K_{\text{eff}}\right)^{\text{calc}}_{B}, \quad (7)$$

where

$$\left(\delta K_{\text{eff}}\right)^{\text{calc}}_{B} = \frac{<\phi_o^*, \delta A_B \phi_o^* >}{<\phi_o^*, K_{\text{eff}}^{-1} \chi S_{f,o} \phi_o^* >}. \quad (8)$$

$\phi_o^*$ being the standard adjoint flux and $\delta A_B$ the perturbation relevant to the control rod insertion.
Likewise, the source reactivity $\rho_{\text{source}}^{\exp}$, associated with a given measured change $\delta s_n^{\exp}$, could be assumed as:

$$\rho_{\text{source}}^{\exp} = \frac{\langle n_{s,o}^*, \delta s_n^{\exp} \rangle}{\langle n_{s,o}^*, \chi S_{f,o} \phi_o \rangle}.$$ 

(9)

Recalling the definition of importance, we obtain, assuming that the perturbation of the source corresponds to a (measured) fractional change of its strength, represented by $\delta s_n^{\exp}/s_n$,

$$\rho_{\text{source}}^{\exp} = \frac{\delta s_n^{\exp}}{s_n} \frac{1}{\langle n_{s,o}^*, \chi S_{f,o} \phi_o \rangle} \equiv \frac{\delta s_n^{\exp}}{s_n} \frac{1 - K_{\text{sub}}}{K_{\text{sub}}}.$$ 

(10)

If we consider changes of the control rod and of the external source, such that the power level remains unaffected, we may write, considering Eqs. (1) and (2) at steady state conditions,

$$\rho_{\text{gen,B}}^{\exp} + \rho_{\text{source}}^{\exp} = 0.$$ 

(11)

Substituting expressions (7) and (10), we have finally

$$\left[ K_{\text{sub}} \rho_{\text{gen,B}}^{\text{cal}} \frac{\delta K_{\text{eff}}}{\delta K_{\text{eff}} / K_{\text{eff}}} \right]_{B}^{\text{cal}} \left( \frac{\delta K_{\text{eff}}}{K_{\text{eff}}} \right)_{B}^{\exp} \frac{1}{1 - K_{\text{sub}}} + \frac{\delta s_n^{\exp}}{s_n} = 0.$$ 

(12)

where, for consistency [cfr. Eqs. (3) and (8)], we have associated coefficient $K_{\text{sub}}$ to $\rho_{\text{gen,B}}^{\text{cal}}$. So, by properly adjusting the external source strength for compensating a control rod insertion, the subcriticality index $(1 - K_{\text{sub}})$ can be estimated. The adjustment could be effected gradually at steps, so that the overall power would keep practically unaltered.

For a reactor at conditions close to critical, at which $K_{\text{sub}} \approx K_{\text{eff}}$ and $n_{s,o}^* = \phi_o^*$, the above expression would simplify into

$$\left( \frac{\delta K_{\text{eff}}}{K_{\text{eff}}} \right)_{B}^{\exp} \frac{1}{1 - K_{\text{eff}}} + \frac{\delta s_n^{\exp}}{s_n} = 0.$$ 

(13)

In this case, the subcriticality level would result

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*This expression for the source reactivity represents a generalization of that defined by Greenspan (1976) at conditions close to criticality, at which function $n_{s,o}^*$ may be substituted by the standard adjoint flux $\phi_o^*$. 

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\[ 1 - K_{\text{eff}} = \frac{(\delta K_{\text{eff}} / K_{\text{eff}})^{\text{exp}}}{\delta S_{n,\text{exp}} / S_{n}}. \]  

(14)

Considering for example an ADS system with \( K_{\text{eff}} = 0.97 \), a control rod insertion corresponding to -0.5% \( \delta K/K \) would imply a 16% source strength increase for maintaining the same power. Considering that the accuracy by which the power level can be estimated in nuclear reactors via outcore fission detectors may reach values of the order of 1%, that would also represent the accuracy associated with the external source adjustment for the power resetting. The uncertainty associated with the subcriticality estimation would then, in this case, not exceed values of the order of 20 pcm. It is interesting to note that the more the subcriticality reduces (the system approaching criticality), the more accurate would become its evaluation, due to the increased source strength change required for compensating the same control rod insertion.

**Asymptotic Power**

The asymptotic power following the insertion of a perturbation which maintains the system subcritical may be obtained from equations (A.16) and (A.17). Setting the time derivatives equal to zero and substituting \( \xi_i = \frac{\beta_i}{\lambda_i} \), it is

\[
\rho_{\text{gen}} P + \xi (1 - P) + \rho_{\text{source}} = 0
\]

(15)

and then, recalling equation (A.24),

\[
P = \frac{1 - K_{\text{sub}} + K_{\text{sub}} \rho_{\text{source}}}{1 - K_{\text{sub}} - K_{\text{sub}} \rho_{\text{gen}}}. 
\]

(16)

As expected, \( P \) increases with \( \rho_{\text{source}} \) (i.e., with increasing source), and with \( \rho_{\text{gen}} \) (i.e., with positive reactivity). At the limit, for \( \rho_{\text{gen}} = \frac{1 - K_{\text{sub}}}{K_{\text{sub}}} \), i.e., with the system approaching criticality, \( P \) diverges.

In order to verify equation (16), we write it in the form, recalling definition (A24),

\[
W = \frac{1 - K_{\text{sub}} + K_{\text{sub}} \rho_{\text{source}}}{1 - K_{\text{sub}} - K_{\text{sub}} \rho_{\text{gen}}} W_0 (1 + q) = \frac{1 + \frac{1}{\xi} \rho_{\text{source}}}{1 - \frac{1}{\xi} \rho_{\text{gen}}} W_0 (1 + q). 
\]

(17)

Recalling definitions (A21) and (A22), and, setting \( s_{o}^* = \gamma \Sigma_{f.o} / W_0 \), that \( <n_{s.o}^*,s_{n.o}^*> = <s_{o}^*,\phi_o^*> = 1 \), we have
\[ W = \gamma \frac{\langle n_{s.o}, (s_{n.o} + \delta s_n) > \Sigma_t \phi_o \rangle (1+q)}{< s^*_o \phi_o > - < n^*_{s.o}, \delta B \phi_o >} \]  \hspace{1cm} (18) 

Since it is \( < s^*_o \phi_o > = - < n^*_{s.o}, B_o \phi_o > \), replacing in numerator and denominator \( \phi \) in place of the unperturbed flux \( \phi_o \) we finally obtain

\[ W = \gamma \frac{\langle n^*_{s.o}, s_n > (1+q) < \Sigma_t \phi_o \rangle}{- < n^*_{s.o}, B \phi_o >} \equiv \gamma \frac{\langle n^*_{s.o}, s_n >}{- < n^*_{s.o}, B \phi_o >} < \Sigma_t \phi_o > \]  \hspace{1cm} (19) 

as we wanted to demonstrate.

**Appendix**

Recently (Gandini, 2001) a derivation was shown of methods for the analysis of subcritical systems, basing on previous results (Gandini, 1997). Here a short (revised) presentation is given.

**Neutron/nuclide evolution**

Considering a system controlled by an external neutron source, the equations relevant to the evolution of the neutron density \( n \), the nuclide density \( c \) and the (intensive) control function \( \rho \), may be written, with obvious notation, in a multigroup neutron energy scheme,

\[ \frac{\partial n}{\partial t} = Bn + \bar{n}s_n \]  \hspace{1cm} (A.1) 

\[ \frac{\partial c}{\partial t} = Ec + s_c \]  \hspace{1cm} (A.2) 

\[ W = < c, Sn > , \]  \hspace{1cm} (A.3) 

\( W \) being the overall system power and \( S \) a matrix with elements containing the microscopic fission cross-sections of the fuel elements.

Consider the following response:

\[ Q = \rho(t_F) \equiv \int_{t_0}^{t_F} \delta(t - t_F) \rho(t) dt \]  \hspace{1cm} (A.4) 

which corresponds to the relative source strength required at \( t_F \) to assure the power level imposed. It may be assumed that, at unperturbed conditions, \( \rho(t) = 1 \). If a parameter (for instance, the initial enrichment, or some other material density) is altered, as in an optimization search analysis, the corresponding change of \( \rho \), and, then, of the external source level at \( t_F \), may be estimated.

Along the HGP T methodology, the sensitivity coefficient relevant to the \( k \)'th generic system parameter \( p_k \) is found as
\[
\frac{\partial p(t_F)}{\partial p_k} = \rho_F^* [\langle n_F^* \frac{\partial}{\partial p_k} (Bn + s_n) \rangle + \langle c, Sn, -W \rangle]_{t_F} \\
+ \int_{t_o}^{t_F} [\langle \tilde{n}^*, \frac{\partial}{\partial p_k} (Bn + s_n) \rangle + \langle c^*, \frac{\partial E}{\partial p_k} c \rangle + \tilde{\rho}^* \frac{\partial}{\partial p_k} (c, Sn, -W)]dt
\]

(A.5)

where \( \rho_F^* = -1/W \), and where \( \tilde{n}_F^*, \tilde{n}^*, c^* \) and \( \tilde{\rho}^* \) are importances satisfying equations

\[
B^* \tilde{n}_F^* + S^T c(t_F) = 0
\]

(A.6)

\[
-\frac{\partial \tilde{n}^*}{\partial t} = B^* \tilde{n}^* + \Omega \tilde{c}^* + S^T \tilde{c}^* \quad t_o \leq t < t_F
\]

(A.7)

\[
-\frac{\partial c^*}{\partial t} = E^* c^* + \Omega^* \tilde{n}^* + S \tilde{\rho}^* \quad t_o \leq t < t_F
\]

(A.8)

and the orthogonality condition

\[
\langle \tilde{n}^*, s \rangle = 0 .
\]

(A.9)

**Stationary case**

At stationary conditions, the sensitivity coefficient given by Eq. (A.5) is simplified into

\[
\frac{\partial p_o}{\partial p_k} = \rho_o^* [\langle n_o^* \frac{\partial}{\partial p_k} (B_{n_o} + s_{n,o}) \rangle + \langle c_o, Sn_o, -W \rangle],
\]

(A.10)

where \( \rho_o^* = -1/W \). The importance function \( n_o^* \) obeys equation

\[
B^* n_o^* + \gamma \Sigma_{f,o} = 0 ,
\]

(A.11)

with \( \gamma \Sigma_{f,o} = S^T c_o \), \( \gamma \) being the number of energy units per fission. To note that \( n_o^* \) corresponds to the importance relevant to the overall power.

**Point kinetic equations**

Let us now consider the equations governing the neutron flux \( \phi (\equiv Vn) \) and precursor \( m_i \) (\( i=1,2,...,I \)) at transient conditions, in a multigroup (\( G \) groups) neutron energy scheme:
\[
V^{-1} \frac{d\phi}{dt} = A\phi + (1 - \beta)\chi_ws_1 \phi + \chi_D u \sum_{i=1}^1 \lambda_i m_i + s_n
\]  
(A.12)

\[
\frac{dm_i}{dt} = \beta_i \chi^T_s \phi - \lambda_i m_i
\]  
(A.13)

where \(A\) is the transport, capture and scattering matrix operator, \(V\) the diagonal neutron velocity matrix, \(u\) is a unit \((G\) component\) vector and

\[
S_f = \begin{bmatrix}
\nu \Sigma_{f,1} & \ldots & \nu \Sigma_{f,G} \\
\ldots & \ldots & \ldots \\
\nu \Sigma_{f,1} & \ldots & \nu \Sigma_{f,G}
\end{bmatrix}
\]

\[
\hat{O}_f = \begin{bmatrix}
\Sigma_{f,1} & \ldots & \Sigma_{f,G}
\end{bmatrix}
\]

\(\chi = \text{diag} \begin{bmatrix} \chi_{z,1} & \ldots & \chi_{z,G} \end{bmatrix}\).

We assume that the transient is initiated at a given time by a perturbation generally changing the unperturbed operators \(A_o, S_{o,i},\) and the neutron source \(s_{n,o}\) into \(A (= A_o + \delta A), S_f (= S_{f,o} + \delta S_f), s_n (= s_{n,o} + \delta s_n),\) respectively, and, consequently, the unperturbed neutron flux \(\phi_o\) into \(\phi (= \phi_o + \delta \phi).\)

Since we want to arrive to an expression accommodating the generalized reactivity expression as defined previously, we introduce then the importance \(n_{s,o}^*\), governed by the equation

\[
A_o^* n_{s,o}^* + S_{f,o}^T [(1 - \beta)\chi_p + \beta \chi_D] n_{s,o}^* + \frac{\gamma}{W_o} \Sigma_{f,o}^* = 0
\]  
(A.14)

similar to Eq. (A.11), but with the external source term normalized to unit power.

The importance \(m_{s,i,o}^*\) associated with the precursor density \(m_{s,o}\) results,

\[
m_{s,i,o}^* \equiv m_{s,o}^* = u^T \chi_D n_{s,o}^*.
\]  
(A.15)

Multiplying Eqs. (A.12) and (A.13) on the left by \(n_{s,o}^T\) and \(m_{s,o}^*\), respectively, space-integrating, after some algebra we obtain, assuming as unperturbed the neutron flux appearing in ratios, and taking into consideration the general dependence of the delayed neutron distribution on nuclide species and energy,

\[
\ell_{eff} \frac{dP}{dt} = (\rho_{gen} - \alpha \beta_{eff}) P + \alpha \sum_{i=1}^1 \lambda_i \xi_i + \zeta (1 - P) + \rho_{source}
\]  
(A.16)

\[
\frac{d\xi_i}{dt} = \beta_{i,eff} P - \lambda_i \xi_i
\]  
(A.17)

where the coefficients represent physically significant quantities, with expressions:
\[ P(t) = \frac{W(t)}{W_0(1 + q)} \left( q = \frac{\langle \delta \Sigma_{f} \cdot \phi_0 \rangle}{\langle \Sigma_{f,o} \cdot \phi_0 \rangle} \right) \] (normalized power) \hfill (A.18)

\[ \xi_i = \frac{\langle n_{s,o}^* m_i \rangle}{\langle n_{s,o}^* \chi S_{f,o} \phi_0 \rangle} \] (i'th effective precursor density) \hfill (A.19)

\[ \ell_{\text{eff}} = \frac{\langle n_{s,o}^* \chi S_{f,o} \rho \phi \rangle}{\langle n_{s,o}^* \chi S_{f,o} \phi_0 \rangle} \] (effective prompt neutron lifetime) \hfill (A.20)

\[ \rho_{\text{gen}} = \frac{\langle n_{s,o}^* (\delta \lambda + \chi \delta S_f) q \phi_0 \rangle}{\langle n_{s,o}^* \chi S_{f,o} \phi_0 \rangle} \] (generalized reactivity) \hfill (A.21)

\[ \rho_{\text{source}} = \frac{\langle n_{s,o}^* \chi S_{f,o} \phi_0 \rangle}{\langle n_{s,o}^* \chi S_{f,o} \phi_0 \rangle} \] (source reactivity) \hfill (A.22)

\[ \alpha = \frac{\langle n_{s,o}^* \chi S_{f,o} \phi_0 \rangle}{\langle n_{s,o}^* \chi S_{f,o} \phi_0 \rangle} \] \hfill (A.23)

\[ \zeta = \frac{1}{\langle n_{s,o}^* \chi S_{f,o} \phi_0 \rangle} \equiv \frac{1 - K_{\text{sub}}}{K_{\text{sub}}} \] \hfill (A.24)

\[ 
\chi \equiv (1 - \beta) \chi_p + \beta \chi_D
\] \hfill (A.25)

\[ \beta_{1,\text{eff}} = \frac{\sum_{g=1}^{G} \sum_{j=1}^{J} \langle n_{s,o,g}^* \chi_{D,g}^j \phi_g \rangle \beta_{i,g}^j \nu \sigma_{f,g}^j \phi_g >}{\sum_{g=1}^{G} \sum_{j=1}^{J} \sum_{i=1}^{I} \langle n_{s,o,g}^* \chi_{D,g}^j \phi_g \rangle \nu \sigma_{f,g}^j \phi_g >}, \quad \beta_{\text{eff}} = \sum_{i=1}^{I} \beta_{i,\text{eff}} \] \hfill (A.26)

and with \( P = P_0 = 1 \) and \( \xi_i = \beta_{i,\text{eff}} / \lambda_i \) at steady state conditions.
REFERENCES

